Challenge 4 involves the computer solution on **two coupled differential equation** to model a population cycle of foxes and rabbits using the **Lotka-Volterra** model. The mathematical solution of the Lotka-Volterra prey-predator system is complex but has been solved mathematicians. However, with the aid of a computer these equations are simple to solve. What's required is a visualization of the model, a rudimentary knowledge of mathematics and the ability to program using any computer language.

Challenge 4: Population Cycle Simulation

A model of population oscillations in a prey-predator system was developed by Lotka and Volterra in the 1920s, and is still very useful today. It can be applied to many prey-predator systems. Challenge 4 uses rabbits and foxes, where x and y are the number of rabbits and foxes respectively at any instant of time.

The Lotka-Volterra model may be **visualized** as follows:

$$a + x \Rightarrow 2x$$

$$x + y \Rightarrow 2y$$

$$y \Rightarrow d$$

Here, *a* represents the (constant) food supply for the rabbits whose population x increases in the first step. The foxes' prey on the rabbits to increase the population y in step two. Finally, when the rabbit population has been reduced sufficiently, and the foxes (which are assumed to eat only rabbits) have become too numerous, they begin to die off in step three for lack of food. This makes it possible for the rabbit population to increase again via step 1, resulting in a repetitive population cycles.

The change in populations x and y in the Lotka-Volterra model can be expressed in two differential equations of the form shown below, where k_1, k_2, k_3 are constants:

$$\frac{dx}{dt} = (k_1 - k_2 y)x \text{ -----(1)}$$

$$\frac{dy}{dt} = (k_2 x - k_3)y \text{ ------(2)}$$

In these equations, k_1 influencing the rate of increase of rabbits over time which is diminished by the term proportional to the number of foxes k_2y. In the second equation, the change in population of the foxes is proportional to the number of rabbits k_2x diminished by the death rate k_3 of the foxes.

The two Lotka-Volterra differential equations may be solved by means of a computer as follows:

Step 1: Translate the given differential equations (the derivative, $\frac{dx}{dt}$, can be treated as a ratio for small increments) to the form:

$dx = (k_1 - k_2y)xdt;$

$dy = (k_2y - k_3)ydt;$

Step 2: Define initial population and constants:
$x=600$, $y=260$, $k_1=0.4$, $k_2=0.004$, $k_3=1.00$, $dt=0.001$

Step 3: Compute the values dx and dy from step 1:

Step 4: Add the increments dx and dy to the starting values of x and y to obtain the new population

x = x + dx;
y = y + dy;

Step 5: Repeat step 3 and step 4 for n iterations see the change in rabbit (x)-fox(y) population over time.

Iterations	No. Rabbits	No. Foxes
0	600	260
100	559	296
200	513	332
300	465	366
400	415	395
500	367	417

Step 6: Decide how to display your data. Table, moving bar graph, time plot, other
Step 7: With the values given is the population (x, y) fluctuating and stable?
Step 8: Find values of constants to produce (a) a dying population, (b) rapid cycles?
Step 9: Share/present your program

Happy Coding, Jim