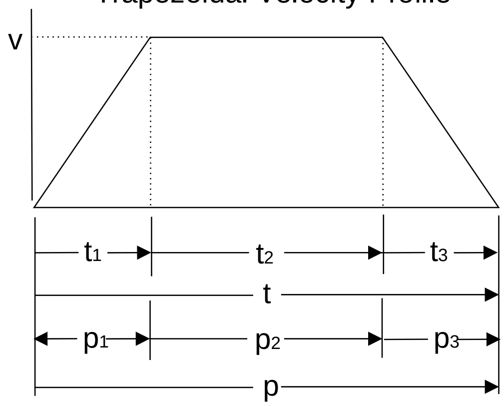
Trapezoidal Velocity Profile



since $t_1 = t_3$ & $p_1 = p_3$, then $t = 2t_1 + t_2$, $p = 2p_1 + p_2$

a: acceleration cm/sec^2

v: velocity in cm/sec after t_1 seconds

t_1: time travelling with constant acceleration a

t_2 time travelling with constant velocity v

t_3 time travelling with constant deceleration a

p_1, p_2, p_3 distances covered in cm

p: total distance travelled

Solution of profile:

Note: a, v and t are known quantities

$p_1 = \frac{1}{2} a t_1^2$ ----(1)

$p_2 = v t_2$ ------(2)

$p = 2p_1 + p_2$, using (1) and (2)

$p = a t_1^2 + v t_2$ -------(3), substituting $t_2 = t - 2t_1$ in (3) gives

$p = a t_1^2 + v(t - 2t_1)$

$p = a t_1^2 + vt - 2vt_1$ ------(4), rearranging (3) gives

$a t_1^2 - 2vt_1 + (vt - p) = 0$, a quadratic equation in t_1

The general quadratic $ax^2 + bx + c = 0$ has solution $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, comparing coefficients gives

$t_1 = \frac{2v \pm \sqrt{(-2v)^2 - 4a(vt - d)}}{2a}$

$t_1 = \frac{v \pm \sqrt{v^2 - a(vt - d)}}{a}$, (numerator/denominator has been divided by 2)

$t_2 = t - 2t_1$

Note:

(1) $if\ v^2 - a(vt - p) < 0, profile\ does\ nor\ exist$ (roots are complex)
(2) If a profile exists, it’s unique- only one profile