## Challenge 2

A hypothetical CNC machine with one axis of travel has three registers labelled as RegP, RegM and RegS, where RegP \& RegM are 16-bits and RegS is 8-bts. Data in Reg $P$ and RegM define the type of motion a cutting tool is to follow. RegS indicates errors in position, velocity, acceleration or time.


Position, velocity, acceleration, and time define a motion. RegS indicates errors.
$0<$ position<=255, final position to move to starting from rest in cm (8-bits)
$0<$ velocity $<=255$, in $\mathrm{cm} / \mathrm{sec}$ ( 8 -bits)
$0<$ acceleration $<=15$, in $\mathrm{cm} / \sec ^{2}$ (4-bits)
$0<$ time $<=15$, total time of travel in seconds (4-bits)
$0<=\boldsymbol{s}<=15$, value of the status register indicating errors
Note:
(1) set $\mathbf{b n}$-> bit bn is replaced by $\mathbf{1}$, reset bn -> bit bn is replaced by 0
(2) Registers data is to be transferred to program 8-bits variables $p, v, a$, and $t$
(3) Errors in $\mathbf{p}, \mathbf{v}, \mathbf{a}$, or $\mathbf{t}$ are to be corrected using the following rules:

Error in $\mathbf{p}$ : reset b0 of $p$
Error in v: replace v by bitwise NOT v
Error in a: set b3 of a
Error in t : exchange bits b0 and b1 of t

## Program Output:

(1) Values comments in brackets are optional
(2) Use the same names for variables described above - helps with discussion
(3) Keeping score? Two points for each output.
(4) Input Data: 33009,9002,11 (initial values - RegP, RegM, RegS)
(5) Final Test Data: End of November

Output \#1: 241 (value of $p$, transferred from RegP)
Output \#2: 186 (value of $v$, transferred from RegM)
Output \#3: 10 (value of a, transferred from RegM)
Output \#4: 220 (value of t , transferred RegM)

Output \# 5 to \#8 - values of $\mathrm{p}, \mathrm{v}, \mathrm{a}$, and t with corrections

Output \#5: 240 (p)
Output \#6: 35 (v)
Output \#7: 10 (a)
Output \#8: 9 (t)

Output \#9: 6.63, 66.33 (time, velocity) see note (1)
Output \#10: 1.32, 7.68 (t1, t2) or "no solution" - see note (2)

Notes:
(1) In Output \# 9 the tool moves, with no profile, from rest to position $p$ with acceleration $\mathrm{a} .6 .6(\mathrm{sec})$ is the time taken for the tool to reach position $p$ and $66.33(\mathrm{~cm} / \mathrm{sec})$ is the velocity at that point in time.
(2)

since $t_{1}=t_{3} \& p_{1}=p_{3}$, then $t=2 t_{1}+t_{2}, p=2 p_{1}+p_{2}$ a: acceleration $\mathrm{cm} / \mathrm{sec}^{2}$
v : velocity in $\mathrm{cm} / \mathrm{sec}$ after t 1 seconds
$\mathrm{t}_{1}$ : time travelling with constant acceleration a $\mathrm{t}_{2}$ time travelling with constant velocity v
ts time travelling with constant deceleration a p1, p2, p3 distances covered in cm p : total distance travelled

In Output \#10 the tool is to move from rest to final position, $\mathbf{p}$, using the trapezoidal "velocity profile" defined by the parameters $p, v$, $a$ and $t$. Movement may not be possible if $p, v$, a and $t$ define a non existing tool path.

The starting point is to solve for $\mathbf{t 1}$ (use the starting equations shown below). The result will be a quadratic equation in $t 1$ of the form: $a x^{2}+b x+c=0$ whose solution is $x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$

Substituting $p, v$, a and $t$ will yields either two values for $t 1$ or no values for $t 1$ (roots are complex- not real numbers).

In Output \#10,
(a) if t1, t2 do not exist output "no solution"
(b) if t1, t2 each have two values (indicting two mathematical tool paths), one set of $\mathrm{t} 1, \mathrm{t} 2$ will be inadmissible. Output the admissible values of $\mathrm{t} 1, \mathrm{t} 2$.

Bonus! Output \#11: phrase or statement (necessary condition for the inadmissible set of $t 1$, t 2 in part (b) to be a tool path)

## Solution of profile:

In the profile $\mathbf{a}, \mathbf{v}, \mathbf{p}$ and $\mathbf{t}$ are positive known (given) quantities leaving $\mathrm{t}_{1}$ and $\mathrm{t}_{2}$ as unknown. Solving for first for $t_{1}$ :

$$
\begin{aligned}
& p_{1}=1 / 2 a t_{1}^{2}-\cdots--(1), \text { where } t 1>0 \\
& p_{2}=v t_{2}-----(2), \text { where } t 2>0, p=2 p_{1}+p_{2}, t=2 t_{1}+t_{2}
\end{aligned}
$$

